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A Linear Programming-Based Methodology to Determine the Minimal Neutron Source Distribution in Subcritical Systems

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ABSTRACT

In this work, we present a linear programming-based methodology to determine the minimum intensity that a neutron source distribution must have to drive a subcritical system to a prescribed power distribution. The neutron source distribution simulates the neutrons produced by spallation events in an accelerator-driven subcritical core. Numerical results are given to illustrate that the offered methodology applied can be used as the initial step of a supplementary computational tool to be used in the design of subcritical nuclear reactors.

KEYWORDS: Subcritical systems, neutron transport, adjoint formulation, optimization problems, linear programming

1. INTRODUCTION

The concept of Accelerator Driven Subcritical (ADS) reactors is based on the idea of coupling a fission nuclear reactor with a high-energy particle accelerator [1]. The ADS reactor core is characterized as subcritical, which means that the fission events occurring within the reactor core fail to release a sufficient number of neutrons to sustain an ongoing series of reactions. The neutronic stability of the reactor core is achieved by an increase in neutron production through the occurrence of spallation nuclear reactions [2]. In short, heavy metal targets are placed inside the reactor core, and the particle accelerator bombards these targets with high-energy protons, releasing tens of neutrons in the target area. These neutrons can eventually migrate to the adjacent fuel areas inducing new fission events, hence sustaining the fission chain reaction in the reactor core.

The ADS reactor has interesting features, such as its increased safety concerning the risk of criticality accidents. As the reactor core is subcritical, one just needs to switch off the accelerator to shut down the reactor [3]. Moreover, the core subcriticality also adds to the ADS reactor transmutation possibilities and the capability to incinerate heavy actinides, especially the fission fuel discarded from commercial nuclear reactors [1]. However, these positive features come with a cost that has to do with the increase in complexity that the need for a particle accelerator adds. The particle accelerator needs to be reliable, i.e., to generate a steady-state proton flux, and be able to provide sufficient power to drive the reactor. Although several important advances have been made concerning particle accelerators, there is more work to do before ADS facilities can be brought to life.

In this work, we present the description of a linear programming-based methodology to determine the minimal source distribution needed to drive a subcritical system to a prescribed power distribution. This preliminary study is the first step in the generation of a supplementary computational tool that can be used in the design process of subcritical nuclear reactors. This tool aims to find, in a design space related to the

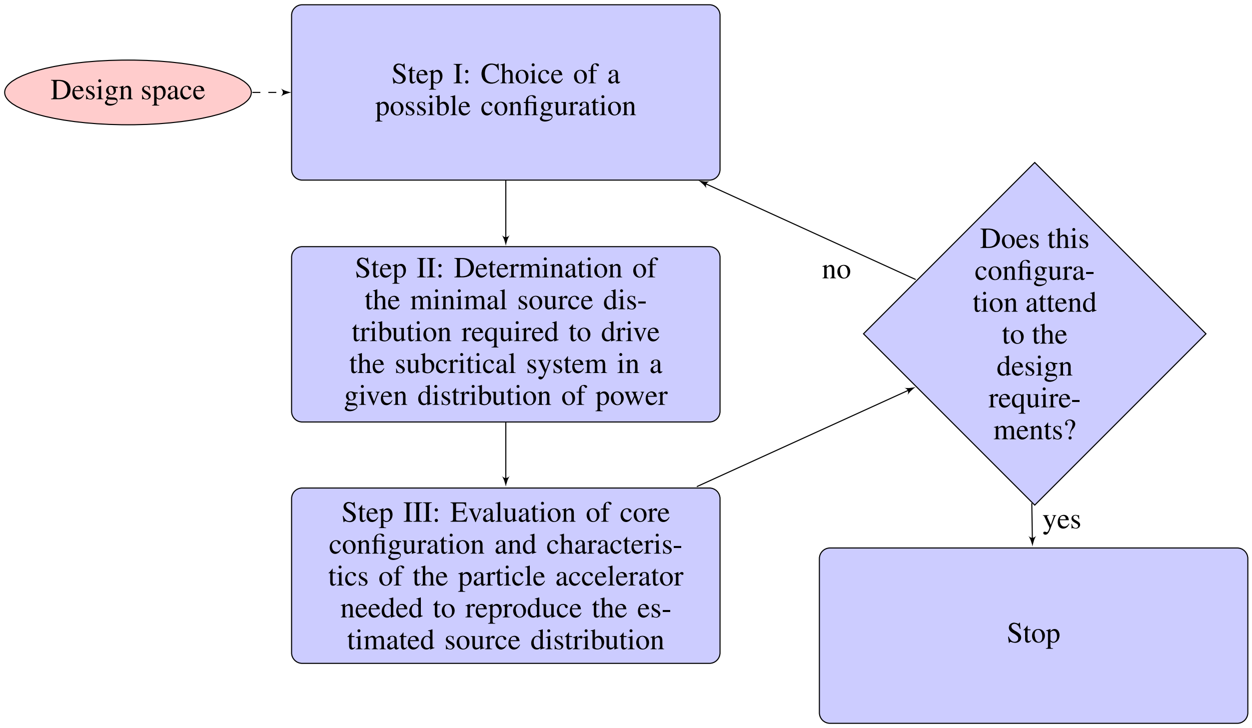


Figure 1: Initial schematic configuration of a supplementary computational tool that can be used in the design of subcritical nuclear reactors.

subcritical core, a core configuration that will decrease the dependence on the particle accelerator. As the development of affordable and reliable particle accelerators that can achieve high values for the beam power is still a challenging ongoing topic of research, the research of potential subcritical core configurations that can decrease the complexity of the needed particle accelerator is particularly interesting.

An initial schematic configuration of the supplementary computational tool operation is given in Fig. 1. The process presented in Fig. 1 stops when a configuration that meets the design requirements is found. Nonetheless, it can be extended to find and compare multiple possible configurations that attend to the design requirements. Analyzing the scheme in Fig. 1, we can notice that the viability of the computational tool depends on the design requirements and efficiencies of the methods that composed steps I-III. At this point, we remark that as this work is an initial step in the supplementary tool's construction, no efficiency analysis will be drawn. Instead, we focus on offering a methodology to represent step II in Fig. 1. To our knowledge, this is the first time that the adjoint methodology is applied in the context of the optimization of neutron source distributions that drive subcritical systems.

A summary of the remainder of this paper is given next. In Section 2, we present a linear programming-based methodology to determine the minimum neutron source distribution that drives a subcritical core to a prescribed power density. The neutron sources considered in this methodology intend to simulate the neutrons generated by spallation events in the reactor core. Therefore, the generated neutron source distribution gives an overall idea of the required intensity that the particle accelerator needs to produce to drive the reactor to a prescribed power density. In Section 3, we present numerical results for one test problem, and we close this paper with a brief discussion in Section 4.

2. METHODOLOGY

We begin by considering a two-dimensional rectangular domain $\mathcal{D} = \left\{(x, y) \in \mathbb{R}^2 \,|0 \leq x \leq L \text{ and } 0 \leq y \leq H \right\}$ composed of contiguous non-overlapping rectangular regions, i.e., $\mathcal{D} = \mathcal{R}_1 \cup \mathcal{R}_2 \cup \cdots \cup \mathcal{R}_k \cup \cdots \cup \mathcal{R}_{T-1} \cup$

$\mathcal{R}_T$, where T represents the total number of regions. The material parameters are considered spatially uniform inside each region $\mathcal{R}_k$ of the heterogeneous rectangular domain.

Now, let us introduce the functional

$$F = \sum_{k=1}^{T} \int_{h_{\mathcal{R}_k}} \int_{w_{\mathcal{R}_k}} \sum_{g=1}^{G} \sum_{m=1}^{M} Q^{\dagger}_{g}(x,y) \Psi_{m,g}(x,y) \omega_m \, dxdy \equiv \left\langle Q^{\dagger}, \Psi \right\rangle, \tag{1}$$

which is linear with respect to both functions $Q^{\dagger}$ and Ψ. In Eq. (1), $m = 1, 2, \ldots, M$ where $M = N(N+2)/2$ with N being the order of the angular quadrature set considered, and $g = 1, 2, \ldots, G$ with G being the number of energy groups in the multigroup formulation. Moreover, $\Psi_{m,g}(x,y)$ is the neutron angular flux in a point of the domain $\mathcal{D}$, migrating in the discrete direction of motion (μ_m, η_m) with kinetic energy within the energy group g. In addition, $h_{\mathcal{R}_k}$ and $w_{\mathcal{R}_k}$ represent, respectively, the intervals in the y and x-axis where region $\mathcal{R}_k$ is located, and ω_m are the weights of the angular quadrature [4]. The group neutron angular flux satisfies the energy multigroup S_N transport equation [4] within the region $\mathcal{R}_k$, i.e.,

$$\left(\mu_m \frac{\partial}{\partial x} + \eta_m \frac{\partial}{\partial y} + \sigma_{t_{k_g}}\right) \Psi_{m,g}(x,y) = \frac{1}{4} \sum_{g'=1}^{G} \left(\sigma_{s_{k,g' \to g}} + \chi_{k,g} \nu \sigma_{f_{k,g'}}\right) \sum_{n=1}^{M} \Psi_{n,g'}(x,y) \omega_n + Q_{k,g}, \tag{2}$$

where $\sigma_{t_{k_g}}$, $\sigma_{s_{k,g' \to g}}$ and $\sigma_{f_{k,g'}}$ are the total, scattering, and fission macroscopic cross sections, and ν and $\chi_{k,g}$ are respectively the average number of neutrons released in a fission event and fission spectrum. The neutron source $Q_{k,g}$ is considered spatially uniform. Details about the constitution of Eq. (2) can be found in [5].

Back to Eq. (1), $Q^{\dagger}_{g}(x,y)$ is a function called adjoint source whose meaning is tied to the definition of the functional F. Based on the concept of neutron importance [6], we can rewrite the functional F, considering appropriate boundary conditions as

$$F = \sum_{k=1}^{T} \int_{h_{\mathcal{R}_k}} \int_{w_{\mathcal{R}_k}} \sum_{g=1}^{G} \sum_{m=1}^{M} Q_{k,g} \Psi^{\dagger}_{m,g}(x,y) \omega_m \, dxdy \equiv \left\langle Q, \Psi^{\dagger} \right\rangle, \tag{3}$$

where $\Psi^{\dagger}_{m,g}(x,y)$ represents the importance that one neutron inserted in the system in a discrete direction of motion (μ_m, η_m) within the energy group g has to the constitution of F. The function $\Psi^{\dagger}_{m,g}(x,y)$ is also referred to as adjoint angular flux since it is the solution of the equation that is adjoint to Eq. (2). That is,

$$\left(-\mu_m \frac{\partial}{\partial x} - \eta_m \frac{\partial}{\partial y} + \sigma_{t_{k_g}}\right) \Psi^{\dagger}_{m,g}(x,y) = \frac{1}{4} \sum_{g'=1}^{G} \left(\sigma_{s_{k,g \to g'}} + \chi_{k,g'} \nu \sigma_{f_{k,g}}\right) \sum_{n=1}^{M} \Psi^{\dagger}_{n,g'}(x,y) \omega_n + Q^{\dagger}_{g}(x,y). \tag{4}$$

At this point, we remark that we are going to use interchangeably the terms importance function and adjoint angular flux to make references for $\Psi^{\dagger}_{m,g}(x,y)$.

Making use of Eqs. (1) and (3) we can write

$$\left\langle Q^{\dagger}, \Psi \right\rangle = \left\langle Q, \Psi^{\dagger} \right\rangle, \tag{5}$$

which is the well-known reciprocity relation [3].

2.1. LINEAR PROGRAMMING OPTIMIZATION PROBLEM (LPOP)

In the second step of Fig. 1, we need, given a possible configuration of the subcritical system, to determine the minimum intensity of the neutron source distribution that will drive the subcritical core in a prescribed distribution of power. We mathematically model this problem through the following linear programming optimization problem

$$\begin{array}{l}\text{Minimize } \sum\limits_{k=1}^{T}\sum\limits_{g=1}^{G} Q_{k,g} \\ \text{Subjected to:} \\ c_1^{min} \leq \mathcal{P}_1 \leq c_1^{max}; \\ c_2^{min} \leq \mathcal{P}_2 \leq c_2^{max}; \\ \qquad\vdots \\ c_T^{min} \leq \mathcal{P}_T \leq c_T^{max}; \\ Q_{k,g} \geq 0; \end{array} , \tag{6}$$

where c_k^{min} and c_k^{max} define the minimum and maximum possible values for the power generated in the region $\mathcal{R}_k$, i.e., $\mathcal{P}_k$. As we can notice from Eq. (6), some constraints are not, in the first moment, directly correlated with the neutron source distribution. We use the adjoint formulation to generate explicit relations between the neutron source distribution and the power generated in the region $\mathcal{R}_k$.

As stated previously in this section, the definition of $Q^\dagger$ is tied to the meaning of the functional F. Therefore, as we want to correlate the neutron source distribution with the power generated in the system, we consider the functional F as the power generated in the region $\mathcal{R}_{k'}$ by the unit of height. That is,

$$F = \mathcal{P}_{k'} = \sum_{k=1}^{T}\int_{h_{\mathcal{R}_k}}\int_{w_{\mathcal{R}_k}}\sum_{g=1}^{G}\sum_{m=1}^{M} \epsilon\sigma_{f_{k,g}}\delta_{k'}(x,y)\Psi_{m,g}(x,y)\omega_m \, dxdy \equiv \left\langle Q^\dagger, \Psi \right\rangle . \tag{7}$$

Analyzing Eq. (7), we can see that the adjoint source term $Q^\dagger$ is

$$Q_g^\dagger(x,y) = \epsilon\sigma_{f_{k,g}}\delta_{k'}(x,y), \tag{8}$$

where ϵ is the average energy released in a fission event, and $\delta_{k'}(x,y)$ is a function that is 1 if $(x,y) \in \mathcal{R}_{k'}$ and 0 otherwise. Substituting Eqs. (3) and (7) into Eq. (5), we obtain

$$\mathcal{P}_{k'} = \sum_{k=1}^{T}\sum_{g=1}^{G} Q_{k,g}\Phi^{\dagger^{k'}}_{k,g}, \tag{9a}$$

where

$$\Phi^{\dagger^{k'}}_{k,g} = \int_{h_{\mathcal{R}_k}}\int_{w_{\mathcal{R}_k}}\sum_{m=1}^{M}\Psi^\dagger_{m,g}(x,y)\omega_m dxdy. \tag{9b}$$

The neutron importance in Eqs. (9) is obtained through the solution of Eq. (4) with adjoint source given in Eq. (8). Making use of Eqs. (9) we can rewrite the LPOP as

$$\begin{array}{l}\text{Minimize } \sum\limits_{k=1}^{T}\sum\limits_{g=1}^{G} Q_{k,g} \\ \text{Subjected to:} \\ c_1^{min} \leq \sum_{k=1}^{T}\sum_{g=1}^{G} Q_{k,g}\Phi^{\dagger^{1}}_{k,g} \leq c_1^{max}; \\ c_2^{min} \leq \sum_{k=1}^{T}\sum_{g=1}^{G} Q_{k,g}\Phi^{\dagger^{2}}_{k,g} \leq c_2^{max}; \\ \qquad\qquad\vdots \\ c_T^{min} \leq \sum_{k=1}^{T}\sum_{g=1}^{G} Q_{k,g}\Phi^{\dagger^{T}}_{k,g} \leq c_T^{max}; \\ Q_{k,g} \geq 0; \end{array} , \tag{10}$$

Equation (10) can be solved through the well-established simplex method [7].

Mathematically, the solution of the LPOP presented in Eq. (10) determines the neutron source distribution that drives the subcritical system in a distribution of power defined by the constraints of Eq. (10), and whose sum is minimal. However, different interpretations can be drawn through the solution of Eq. (10). We may consider $Q_{k,g}$ as the number of neutrons within energy group g that are released in spallation events in the region $\mathcal{R}_k$. In this case, the neutron source distribution represents the distribution of neutrons released in the spallation target areas. If we assume that interpretation for the neutron source distribution, we need to ensure that $Q_{k,g}$ is zero in the regions that do not contain spallation material. This can be done by adding the constraint

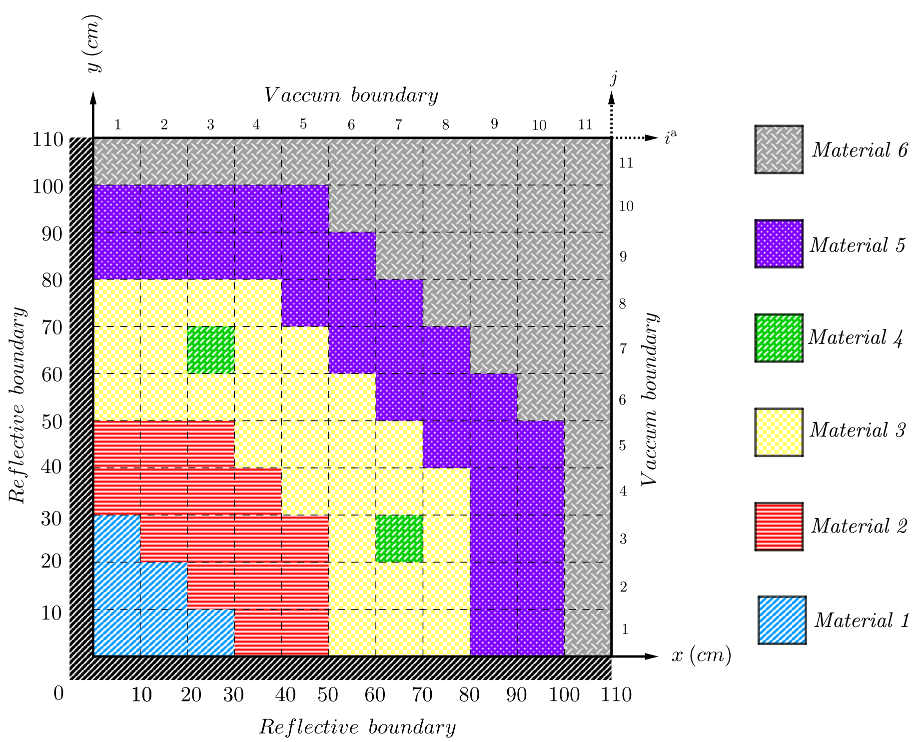
$$Q_{k,g} = 0, \tag{11}$$

in the LPOP for each k different from the areas with spallation material and g are the energy groups different from the energy groups of the neutrons released in spallation events.

Another possible interpretation for the solution of Eq. (10) is to consider $Q_{k,g}$ as the number of neutrons within energy group g that arrive region $\mathcal{R}_k$ from spallation events produced in a region different than $\mathcal{R}_k$. Look that this interpretation is entirely different from the previous one, but can also be useful since the only assumption made is about the influence of the spallation events in areas other than the target areas. To represent this problem, we need to make sure that Eq. (11) is valid for the areas $\mathcal{R}_k$ that contain the spallation material.

3. NUMERICAL RESULTS

In this section, we present numerical results for a Test problem whose geometry and material distribution is illustrated in Fig. 2, with material parameters presented in Table 1. In this problem, we consider that the average number of neutrons released in fission events is $\nu = 2.4$, and the average energy released in fission events is $\epsilon = 200\ MeV$. Additionally, we use the S$_4$ formulation. We remark that this Test problem, as represented in Fig. 2 and Table 1, is fictitious. The idea is to demonstrate the application of the methodology described in the previous section and to show that this methodology can be used as the step II of the supplementary tool whose configuration is illustrated in Fig. 1.



[a] The values of i and j define the region $\mathcal{R}_k$. That is, $k = i + (j-1) \times 11$.

Figure 2: Geometry and material distribution of the Test problem.

Table 1: Material parameters of the Test problem.

Material parameters		σ_{t_g} (cm^{-1})	σ_{f_g} (cm^{-1})	$\sigma_{s_{g\to g'}}$ (cm^{-1})		X_g
				$g'=1$	$g'=2$	
Material 1	$g=1$	2.31800E-01[a]	0.00000E+00	1.82450E-01	1.56300E-02	
	$g=2$	8.38000E-01	0.00000E+00	0.00000E+00	7.33100E-01	
Material 2	$g=1$	2.31800E-01	2.99700E-03	1.82400E-01	1.63800E-02	1
	$g=2$	8.71460E-01	6.53510E-02	0.00000E+00	7.42840E-01	0
Material 3	$g=1$	2.36570E-01	3.68710E-03	2.09560E-01	1.45300E-02	1
	$g=2$	8.22800E-01	8.37530E-02	0.00000E+00	7.06500E-01	0
Material 4	$g=1$	2.71520E-02	0.00000E+00	2.64812E-02	1.63800E-04	
	$g=2$	5.22800E-02	0.00000E+00	0.00000E+00	5.10840E-02	
Material 5	$g=1$	3.37600E-01	0.00000E+00	3.03126E-01	1.01200E-03	
	$g=2$	9.99500E-01	0.00000E+00	0.00000E+00	3.13126E-01	
Material 6	$g=1$	1.78120E-01	0.00000E+00	8.00710E-02	3.43400E-03	
	$g=2$	1.17616E+00	0.00000E+00	0.00000E+00	1.14458E-01	

[a] Read as 2.31800×10^{-01}.

To construct the LPOP given in Eq. (10), we need first to provide the prescribed power distribution. In this case, we assume that the sum of the power generated by the regions of material 2 and 3 should be in the interval $[4\, MW\, cm^{-1},\ 5\, MW\, cm^{-1}]$ each. That is,

$$4\, MW\, cm^{-1} \leq \mathcal{P}_{material\,2} \leq 5\, MW\, cm^{-1}, \tag{12a}$$

and

$$4\, MW\, cm^{-1} \leq \mathcal{P}_{material\,3} \leq 5\, MW\, cm^{-1}, \tag{12b}$$

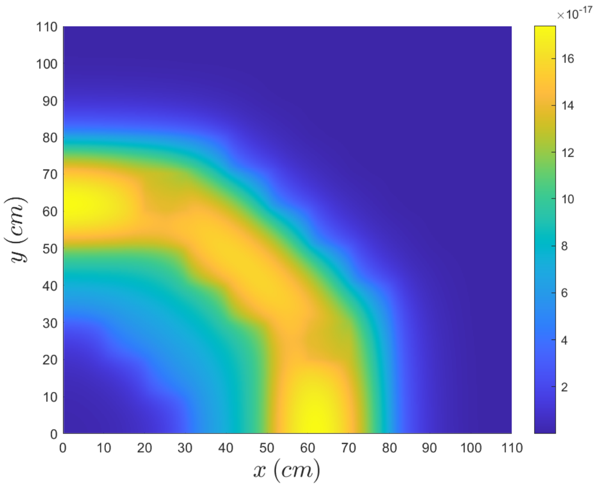
where $\mathcal{P}_{material\,2}$ and $\mathcal{P}_{material\,3}$ represent the total power generated by the regions composed of material 2 and 3, respectively.

Equations (12) are the initial constraints of the LPOP. To associate these constraints to the neutron source distribution, we solve two adjoint problems (Eq. (4)), varying the adjoint source term accordingly. The adjoint source terms associated with Eqs. (12) must be chosen such that the functional presented in Eq. (1) becomes the total power generated by the regions of material 2 and 3, respectively. To solve Eq. (4), we chose to use the coarse-mesh Response Matrix-Constant Nodal (RM-CN) method [3]. To generate accurate results for the importance function making use of the RM-CN method, we discretized the two-dimensional spatial domain in square cells of length equal to $0.2\, cm$. The stopping criteria adopted for the RM method require that the relative deviations between two consecutive estimates of the adjoint node-edge average scalar fluxes be no greater than 1×10^{-6}. Figure 3 displays the group node-average adjoint scalar fluxes as a function of the node midpoints.

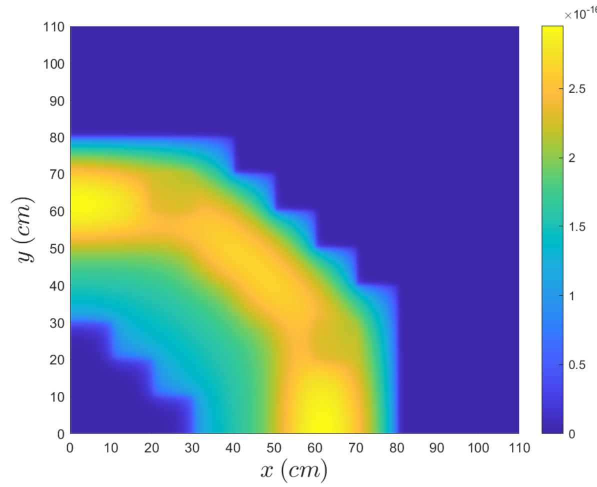
As briefly discussed in the previous section, the solution of the LPOP may have multiple interpretations. Thus, we consider $Q_{_{k,g}}$ as the number of neutrons that are released in spallation events inside the regions $\mathcal{R}_k$ which are composed of spallation material. Assuming that the regions composed of material 1 are the regions where spallation events can occur, we need to add the constraint

$$Q_{_{k,g}} = 0, \tag{13}$$

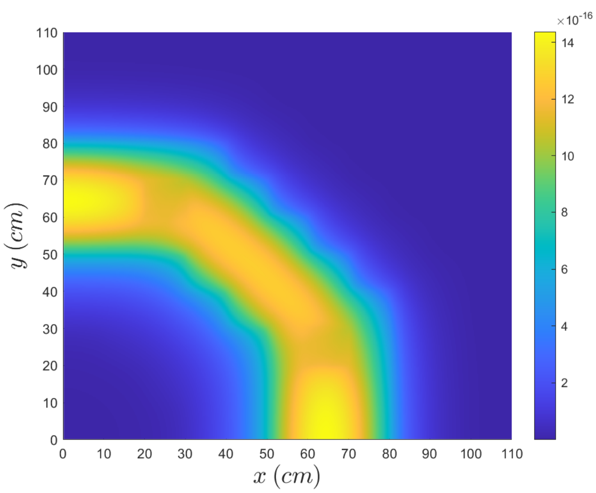
to the LPOP. Equation (13) is valid for all regions composed of a material that is different from material 1. Once all the constraints of Eq. (10) are settled, we solve the LPOP using the simplex method. The



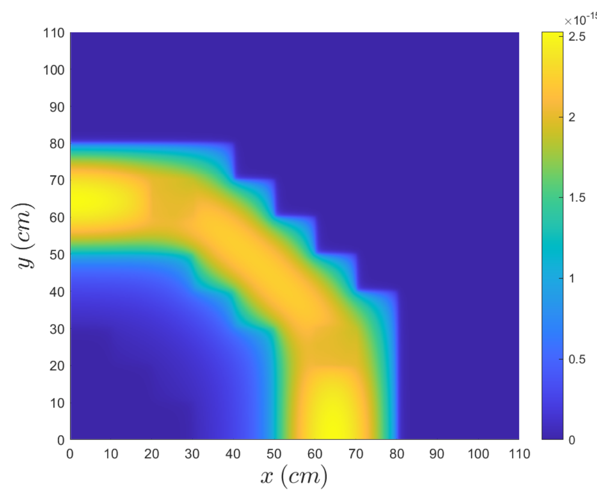
(a) **Generated by the solution of Eq.** (4) **with** $g = 1$**, considering adjoint source term associated to the total power produced by the regions composed of material 2.**



(b) **Generated by the solution of Eq.** (4) **with** $g = 2$**, considering adjoint source term associated to the total power produced by the regions composed of material 2.**



(c) **Generated by the solution of Eq.** (4) **with** $g = 1$**, considering adjoint source term associated to the total power produced by the regions composed of material 3.**



(d) **Generated by the solution of Eq.** (4) **with** $g = 2$**, considering adjoint source term associated to the total power produced by the regions composed of material 3.**

Figure 3: Node-average adjoint scalar fluxes for the LPOP of the Test problem.

solution of the LPOP is $Q_{_{23,1}} = 1.29743E + 15\, neutrons/cm^3 s$, $Q_{_{23,2}} = 5.31066E + 14\, neutrons/cm^3 s$ ($k = 23 \rightarrow i = 1$ and $j = 3$, according to Fig. 2), and $Q_{_{k,g}} = 0$ for the remaining of regions and energy groups.

The solution of the LPOP can be explained by making use of the results displayed in Fig. 3 in conjunction with the values of the material parameters presented in Table 1. In Figs. 3a and 3b, we have results for the importance function associated with the total power generated by the regions composed of material 2, and in Figs. 3c and 3d, we have results for the importance function associated with the total power generated by the regions composed of material 3. In both cases, the neutrons that migrate in the regions of material 3 are more important to the overall generation of power. If we analyze the macroscopic cross sections of materials 2 and 3, we can notice that the fission cross sections of material 3 are higher than the fission cross sections of material 2. This means that neutrons migrating in regions composed of material 3 will induce more fission events than neutrons migrating in regions composed of material 2. Therefore, more neutrons will be released through fission events in the regions composed of material 3 compared to regions of material 2. Some neutrons released in fission events in regions of material 3 will eventually migrate to regions composed of material 2, thus inducing new fission events. For that reason, neutrons migrating in regions composed of material 3 are also important to the generation of power in regions composed of material 2.

Based on this idea, a large number of neutrons in the first energy group (smaller cross sections) should be released in a region composed of material 1, so that a considerable part of it can reach regions composed of material 3. Furthermore, in the path to reach the regions composed of material 3, some neutrons induce fission events in regions composed of material 2, which generates a value of power that is smaller than the prescribed for the regions of material 2. Thus, additional neutrons in the second energy group (higher cross sections) should also be released in spallation target areas, so that the total power generated by the regions composed of material 2 can achieve the prescribed value of power. Finally, the neutron source is located in the region $\mathcal{R}_{_{23}}$ ($i = 1$ and $j = 3$) because neutrons released in this region have to travel a smaller path to reach regions of material 3. This increases the probability of a neutron reaching regions composed of material 3. Thus, compared to other regions composed of material 1, a smaller number of neutrons need to be released in the region $\mathcal{R}_{_{23}}$ to generate the prescribed power considered for the regions of material 3.

Now, to visualize if this source distribution drives the subcritical core to the prescribed power distribution, we solve Eq. (2) with the same parameters of the Test problem using the neutron source distribution generated by the solution of the LPOP. We remark that we also have used the RM-CN method to solve Eq. (2). The results are shown in Table 2.

If we sum the power generated in the regions composed of materials 2 and 3 we obtain, respectively, $\mathcal{P}_{material\,2} = 4.00047\, MW/cm$ and $\mathcal{P}_{material\,3} = 5.00411\, MW/cm$, which ensures that the methodology presented in Section 2 can potentially be used as the main part of step II presented in Fig. 1.

4. DISCUSSION

Here we presented a methodology to estimate the minimal intensity of neutron source distribution that drives a subcritical core to a prescribed power density. That is the initial step of a supplementary computational tool that can be used in the design process of subcritical nuclear reactors. Numerical results were given in the previous section to illustrate the methodology's application, and to show that this methodology can potentially represent step II of Fig. 1.

As an initial study, no efficiency analysis was conducted. However, the computational tool's viability will depend on the viability of the methods chosen to represent steps I to III. Therefore, an efficiency analysis of the methodology is of importance and should await future work. Nonetheless, even though an efficiency analysis needs to be performed, one can argue that the most costly part of the LPOP presented in Eq. (10) resides in the constitution of the constraints associated with the power generated in the system. As currently stated, to generate the constraints of the LPOP, we need to solve one adjoint problem for each value of power that defines the prescribed power distribution.

Different aspects may impact the efficiency concerning the generation of the constraints of the LPOP. Some examples are the constitution of the power distribution; and the fidelity of the mathematical model used to

Table 2: Power generated through the solution of Eq. (2) with the solution of the LPOP.

Power generated ($MW\ cm^{-1}$)											Region $\mathcal{R}_k$[a]
$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	$i=11$	
											$j=11$
											$j=10$
											$j=9$
0.2411022	0.2139447	0.1798498	0.1206185								$j=8$
0.3899762	0.3319572		0.2207724	0.1215108							$j=7$
0.3997266	0.3358114	0.2678758	0.2406692	0.1856140	0.0980773						$j=6$
0.3194759	0.2260351	0.1510796	0.1915111	0.1830440	0.1398101	0.0721037					$j=5$
1.2030886	0.4623903	0.1500690	0.0912097	0.1312615	0.1306511	0.0985042	0.0485739				$j=4$
	0.9217032	0.1494500	0.0582329	0.0562733	0.0993961		0.0638004				$j=3$
		0.0853242	0.0365341	0.0374854	0.0862731	0.0984906	0.0658160				$j=2$
			0.0210569	0.0310708	0.0823938	0.0989203	0.0660577				$j=1$

[a] The values of i and j define the region $\mathcal{R}_k$. That is, $k = i + (j - 1) \times 11$.

represent the adjoint problems. The impact that the prescribed power distribution has on the efficiency of the LPOP resides in the number of adjoint problems that need to be solved. The more restrictions on the power generated by the system are made, the more adjoint problems need to be solved to create the necessary constraints. One idea that can be explored in the future to approach this situation is the generation of auxiliary equations to be used in conjunction with the importance function associated with the total power generated by the system. If we consider the adjoint source term such that the functional given in Eq. (1) is the power generated by the whole system, the solution of Eq. (4) becomes the importance function associated with the total power generated by the system. The adjoint source term is, in this case, composed of the sum of the adjoint source terms associated with the power generated by each prescribed value of power. Therefore, as Eq. (4) is linear and the boundary conditions applied to Eq. (4) are homogeneous, the importance function associated with the total power is equal to the sum of the importance functions associated with the prescribed value of power. Thus, instead of solving Eq. (4) for each prescribed value of power, one could potentially solve Eq. (4) only one time, for the total power generated by the system, and explore auxiliary equations that would allow separating from this importance function the contribution of the importance functions associated with each prescribed value of power.

Another way that may potentially increase the efficiency concerning the constraints of the LPOP is the exploration of low-fidelity models to represent the adjoint problems. Analyzing Eq. (10), we notice that to define the constraints of the LPOP, we need to calculate Eq. (9b) for all regions and energy groups associated with each prescribed value of power. That indicates that we do not necessarily need to know $\Psi^\dagger$ accurately, but instead, we need to calculate $\Phi^\dagger$ accurately. Since the accurate calculation of $\Psi^\dagger$ is not directly needed, the exploration of low fidelity models that can accurately calculate $\Phi^\dagger$ is of interest.

In conjunction with the topics of future work already discussed in this section, we intend to continue this work by developing/applying methods to represent steps I and III of Fig. 1. Regarding steps I and III, the use of search tools/methods that can quickly move towards configurations that attend to the desired requirement is of particular interest.

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